Dependent Events
Coco Wildlife Conservation Trust
Lesson 41-1 Understanding Conditional Probability

Learning Targets:
- Understand the conditional probability of *A* given *B*.
- Determine conditional probabilities using two-way frequency tables and Venn diagrams.
- Interpret the answer in terms of the model.

SUGGESTED LEARNING STRATEGIES: Summarizing, Paraphrasing, Think Aloud, Create Representations, Visualization, Think-Pair-Share, Debriefing

1. There are 90 cranes that belong to the Coco Wildlife Conservation Trust. The cranes have been categorized by gender and type as shown in the two-way frequency table.

	Male	Female	Total
Gray-crowned	16	13	
White-naped	17	14	
Stanley	11	19	
Total			90

a. Complete the table by entering the other totals.

b. If a female crane is selected at random, what is the probability that it is a Stanley crane?

c. Calculate the probability that a crane is a Stanley crane given that it is female.

You found the probability that a crane is a Stanley crane *given that it is female*. This is a ***conditional probability***. We know that the crane is female. Then we find the probability that it is a Stanley crane.

In probability notation we show conditional probabilities using a vertical line. The probability that a crane is a Stanley crane given that it is female is written $P(\text{Stanley} \mid \text{female})$. If we denote the events by their initial letters, this conditional probability is written $P(S \mid F)$. $P(S \mid F)$ is read "the probability of S given F."

MATH TERMS

A **conditional probability** is the probability that an event occurs given that another event occurs.

WRITING MATH

Probability Notation

The probability of A given B is written $P(A \mid B)$.

2. Write the following probabilities using probability notation.
   a. the probability that a crane is male given that it is a gray-crowned crane

   b. the probability that a crane is a gray-crowned crane given that it is male

   c. the probability that a randomly selected white-naped crane is female

Check Your Understanding

3. The Coco Trust also has flamingos. The flamingos are categorized as shown in the table.

	Male	Female	Total
Chilean	16	13	
Greater	17	14	
Total			

   a. Complete the table by entering the totals.
   b. Suppose that a flamingo will be selected at random. The various possible events are denoted by their initial letters. Calculate the following probabilities to the nearest thousandth.
     i. $P(M)$
     ii. $P(M \mid C)$
     iii. $P(C \mid M)$
   c. If a female flamingo is selected at random, to the nearest thousandth, what is the probability that it is a greater flamingo? Use conditional probability notation to write your answer.

4. As part of their math course, Kate and Rob were required to do a statistical study. They selected a random sample of 80 students from their school. They asked all the selected students their grade level (9, 10, 11, or 12) and whether or not they had $5 or more with them. The results are shown in the table.

	9th Grade	10th Grade	11th Grade	12th Grade
Less than $5	13	10	9	3
$5 or more	8	9	17	11
Total				

a. Complete the table by entering the totals.
b. Attend to precision. A student will be selected at random from those in the sample. Find the following probabilities to the nearest hundredth.
  i. *P*($5 or more | 9th grade)

  ii. *P*($5 or more | 10th grade)

  iii. *P*($5 or more | 11th grade)

  iv. *P*($5 or more | 12th grade)

c. Since the sample was selected randomly, the students in the sample are likely to be representative of the school as a whole. What do the answers to Part b suggest to Kate and Rob about the amounts of money carried by students at the school?

My Notes

Check Your Understanding

5. Suppose that a student is selected at random from your school. What would be the probabilities of each of the following events?
  a. the probability that the student plays the trumpet
  b. the probability that the student plays the trumpet given that the student is in the school band

6. Make sense of problems. Let T be the event that the student plays trumpet and let B be the event that the student is in the school band. Explain in words the meaning of each of the following probabilities.
  a. $P(T \cap B)$
  b. $P(T \mid B)$
  c. Are the probabilities in Parts a and b the same? Explain. (*Hint*: Use numbers.)

7. Some of the Trust's animals are tortoises. Some are not. Some of the animals are of known gender. Some are not. Let the set of tortoises be T and the set of animals of known gender be K. The numbers of animals falling into these categories are shown.
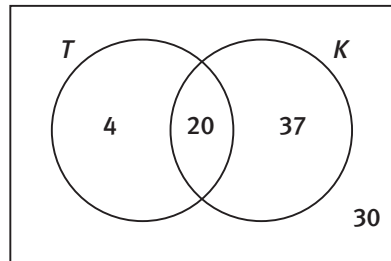
  a. How many animals are there at this location?

  b. How many of these animals are tortoises?

  c. How many of these animals are of known gender?

  d. How many of the animals of known gender are tortoises?

e. Find each of the following probabilities to the nearest thousandth.
Use probability notation to write your answer.
   i. Suppose an animal of known gender is selected at random. What
   is the probability that this animal is a tortoise?

   ii. Calculate $P(K \mid T)$.

   iii. Do $P(T \mid K)$ and $P(K \mid T)$ represent the same region on the Venn
   diagram? Are the probabilities equal? Explain.

   iv. Suppose a tortoise is selected at random. What is the probability
   that its gender is not known?

My Notes

Check Your Understanding

8. Reason quantitatively. Recall that at Annabel High School, some
students take Spanish and some do not. Some students take an art
class, and some do not. Let *S* be the set of students who take Spanish
and *A* be the set of students who take an art class. The numbers of
students in various combinations of these classes are given in the
Venn diagram.
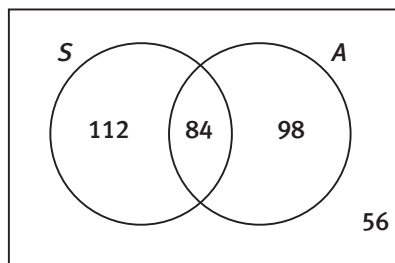
a. If a student is selected at random from the school, what is the
probability that the student takes Spanish?
b. Find $P(S \mid A)$.
c. Find $P(A \mid S)$.
d. Which of the answers to Parts a and b is larger? Explain in your
own words what this means.
e. Find $P(S \cap A \mid S^{C})$.

My Notes

LESSON 41-1 PRACTICE

9. Recall the phone store. Some of the phones have caller ID. Some do not. Some have a speakerphone. Some do not. The numbers of phones with and without these features are shown in the table.
   a. Write the totals in the table.

	Speakerphone	No Speakerphone	Total
Caller ID	18	2	
No Caller ID	16	12	
Total			

Let C be the event that a phone has caller ID and let S be the event that a phone has a speakerphone. Calculate each probability to the nearest thousandth.
   b. If a phone is selected at random from those available at the store, what is the probability that it has a speakerphone?
   c. If you are now told that a phone has caller ID, does this make it more or less likely that it has a speakerphone?
   d. Find $P(C \mid S^C)$.

10. a. Draw a Venn diagram to illustrate the information in the table in Item 9.
    b. If a phone is selected at random from those available at the store, what is the probability that it has caller ID?
    c. If you are now told that a phone has a speakerphone, does this make it more or less likely that it has caller ID?

11. **Model with mathematics.** Recall the SpringBoard Superstar contest. This table shows four categories into which the contestants can be placed.

	Male	Female
Under 20	1	2
20 or Over	5	4

   a. Suppose that a contestant will be selected at random. Find the following probabilities to the nearest thousandth.
      i. $P(\text{female})$
      ii. $P(\text{female} \mid \text{under 20})$
      iii. $P(\text{female} \mid \text{20 or over})$
   b. Suppose you are a reporter for a newspaper. Write an article about the three probabilities you found in Part a. Compare these probabilities.